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Number Freak

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NUMBER FREAK

From 1 to 200

The Hidden Language of Numbers Revealed

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1

THE number 1 is both a logical and a lousy way to start this book. Logical because 1 comes first, and its omission would seem absurd. But also lousy, because this book is about special properties of whole numbers, and the number 1 just has too many special properties for its own good.

For starters, 1 is the "multiplicative identity"—if you multiply any number by 1, that number is left unchanged. In particular, 1 is its own square and cube, and in general 1 to the *n*th power equals 1 for any *n*. A probability of 1 is the same thing as certainty, and 1 is also the maximum value attained by the basic trigonometric functions sine and cosine. The number 1 is also a trivial solution to many equations that mathematicians study, so much so that it pretty much has to be exempted: Not only is it a perfect square and a perfect cube, it's a triangular, pentagonal, and hexagonal number, and so on. You see the problem?

The idea that the number 1 is everywhere has a specific foundation in the form of Benford's Law. Introduced in 1938 by physicist Frank Benford, the idea is that the digits found in a variety of naturally occurring datasets (molecular weights, population sizes, digits on the front pages of newspapers, to name a few of the thousands of sets studied by Benford) are not uniformly distributed. In particular, the leading digit of such numbers is 1 about three-tenths of the time, far above the one-in-nine frequency you might expect. (Sorry, but zeroes don't count in this context.)

The progenitor to Benford's Law was astronomer Simon Newcomb's 1881 discovery that within a book of logarithm tables, the pages containing logarithms beginning with 1 were much more likely to be worn around the edges. More recently, Benford's Law has been applied in the detection of tax and

accounting fraud. The basic principle is that people who make up numbers are not familiar with Benford's Law, and the bogus datasets they generate stand out because they have far too few 1's in them.

THE Greek philosopher Parmenides (fifth century BC) was of the view that "all is one." While I can't say I understand exactly what he meant, apparently it was Parmenides who inspired Zeno (which can be spelled with either a *Z* or an *X*, followed by *one* backward) to come up with his now famous set of paradoxes. Perhaps the best known of these paradoxes, known in the trade as Dichotomy, asserts that before you can ever reach a destination, you have to go halfway first. No problem there, except that from the halfway point you have to go halfway again, and so on, the paradox being that as long as you do so you'll never quite get to your destination. This line of reasoning confounded the thinkers of Zeno's day, but by the time Newton and Leibniz drafted the concepts of calculus in the late 1600s, the notion that an infinite series of numbers could converge to a finite number was no longer paradoxical. The specific series at hand is represented by the equation

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots, \text{ or more compactly as } 1 = \sum_{n=1}^{\infty} \frac{1}{2^n}$$

A more advanced type of paradox involving the number 1 came from the great French mathematician Henri Lebesgue (1875–1941). His Lebesgue measure (pronounced *le-bayg*, by the way) provided a means for measuring various subsets of Euclidean space. A simple example would be the interval [0,1] (namely, all real numbers between 0 and 1), which has a Lebesgue measure of 1. No problem so far, but it jars our intuition when we hear that the set of irrational numbers between 0 and 1 also has measure 1. In other words, the entire set of proper fractions, infinite though it may be, has a measure of zero, a stunning reminder that not all infinities are the same.

Lebesgue was apparently the last notable mathematician to think of 1 as a prime number. Yes, the late astronomer Carl Sagan included 1 as a prime in his 1985 book *Contact*, but nowadays most mathematicians would be more swayed by the views of longtime UCLA professor and Einstein protégé Ernst Gabor Straus, to whom is attributed the following quotation: "The primes are the building bricks of arithmetic, and 1 is just not a brick!" In that spirit, 1 is the only number in this volume that will not be designated at the top of its page as being either prime or composite; in the latter case, we will of course be kind enough to provide the factorization.

IN the world of music, *A Chorus Line* highlighted one as a singular sensation, but Harry Nilsson had already warned us that it is also the loneliest number. Three Dog Night's 1969 version of the Nilsson song "One" was that band's first gold record, although it did not reach number 1 on the Billboard charts.

IN chess, the number 1 is used to indicate victory, so a line-by-line recap of a chess game that ends with 1–0 indicates a victory by white, while $\frac{1}{2}-\frac{1}{2}$ indicates a draw and 0–1 indicates a black victory.

FINALLY, the phrase "We're number one" is by now a rather tedious refrain, especially when it comes from the other side of a football stadium, but the expression isn't about to be replaced, such is the power of one. Along those lines, it must be said that the expression "public enemy number one" simply isn't what it was in the days of Al Capone. Nor is the "identity parade," known to Capone and even law-abiding Americans as the "police lineup," whose popularity has fallen with the advent of computer databases and DNA technology. But even when lineups were in vogue, many police sta-



tions started their lineups with number 2, simply because the mighty number 1 was chosen disproportionately often, no matter where the guilty party happened to be standing. Frank Benford might have predicted as much.

2 [prime]

THE number 2 is the only even prime number. It is also the only prime number that lacks an *e* in its name, but that's because every odd number has an *e*.

THE square root of 2, written $\sqrt{2}$, was the first number shown to be irrational, meaning that it cannot be written as the quotient of two whole numbers. The discovery of this fact is credited to Hippasus of Metapontum (circa 500 BC), and its proof is surprisingly easy. Just assume that $\sqrt{2} = \frac{p}{q}$, with p and q positive integers with no common factors. If we square both sides we get $2 = \frac{p^2}{q^2}$, or $2q^2 = p^2$. But it's not possible for one perfect square to be twice another. Specifically, note that p can't be odd in the above equation, because if it were, the right-hand side would be odd, whereas the left-hand side is clearly even. But if p is even it has a factor of 2, and there p^2 would be divisible by 4. But since $p^2 = 2q^2$, that would mean q is even, violating our assumption that p and q have no common factor. Hence the equation $2q^2 = p^2$ cannot ever hold.

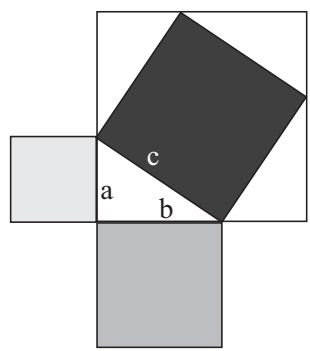
Okay, maybe that wasn't as easy as you would have liked, but don't be jealous of Hippasus. Apparently the notion of irrational numbers was an affront to his fellow Pythagoreans, a group that thought the world revolved around nice simple ratios. Hippasus made his discovery while at sea, or so the legend goes, whereupon his shipmates threw him overboard. Nowadays we are more tolerant of mathematical discovery, although it should be said that those who demonstrate an early aptitude for mathematics are often the object of derision and ridicule in grade school. The irony of the Hippasus tale is that the set of irrational numbers turns out to be a higher-order in-

finity than the set of rational numbers, a fundamental fact that neither he nor his tormentors lived to discover.

THE prefix *bi-* means two, as in bicycle, binoculars, biplane, bisect, and binary. Curiously, the word *biscuit* also applies, as it derives from the Italian biscotti, which literally means "twice cooked."

IN geometry, the number 2 shows up in the famous saying "two points determine a line." This statement is emphatically true in planar geometry, but frequently abused in the real world, where two data points may be insufficient to determine an actual trend.

IT is well-known that if a right triangle has legs a and b and hypotenuse c, then $a^2 + b^2 = c^2$. This equation, known to one and all as the Pythagorean Theorem, may not even be the most recognized equation involving c^2 (that title probably belongs to $E = mc^2$), but it is still way up there on the list of recognizable mathematical equations. And it's a mite easier to prove than Einstein's trademark equation, as the following diagram shows:



The largest square measures $a + b$ on each side, so its area equals $(a + b)^2$. But this square also consists of the square with side c coupled with

four triangles each with area $\frac{ab}{2}$. Putting everything together, $(a + b)^2 = c^2 + 4(\frac{ab}{2})$. Multiplying out the left-hand side, $a^2 + 2ab + b^2 = c^2 + 2ab$, so $a^2 + b^2 = c^2$. In particular, if a and b both equal 1, c equals the square root of 2, and the exploration of what c could possibly be is apparently what got Hippasus his ticket to Davy Jones's locker. (It was fourth-century BC geometer Theaetetus who is credited with the more general result that the square root of any integer that is not a perfect square must be an irrational number.)

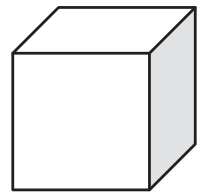
Note that the above proof required that the angle between a and b was a right angle, because otherwise the areas of the white triangles would have been more complicated to calculate. You should know that the Pythagorean Theorem works the other way around—if the legs of a triangle are such that $a^2 + b^2 = c^2$, then the triangle in question is a right triangle. That much was common knowledge in the time of Euclid (300 BC). Since Euclid's day there have been hundreds of published proofs of the Pythagorean Theorem. One of the most notable was devised in 1876 by James A. Garfield, who became president of the United States five years later. (Alas, nowadays anyone with such mathematical curiosity and competence would probably have it used against him in a national election.)

THE Pythagorean Theorem generalizes to three dimensions, so if a, b, and c are the three sides of a rectangular prism (box) and d is its diagonal, you have the equation $a^2 + b^2 + c^2 = d^2$. And it even generalizes in two dimensions to triangles other than right triangles, yielding the law of cosines.

But the one thing you can't do to the Pythagorean Theorem is to take the 2 out of it. The celebrated Fermat's Last Theorem, proved by Princeton's Andrew Wiles in the mid-1990s after a 300-year wild-goose chase, declares that the equation $x^n + y^n = z^n$ has no solutions for any positive integer n other than 2. (Okay, I lied. That equation has infinitely many solutions when $n = 1$, proving the point made in **1** that 1 can be a nuisance, because so many mathematical truths "accidentally" include 1 as a completely trivial case.)

FOR a final appearance of the number 2 in a geometric context, it shows up in a formula derived by the great Swiss mathematician Leonhard Euler (1707–1783), who noticed that in a three-dimensional shape known as a polyhedron, there is a fixed relationship between the number of edges, faces, and vertices.

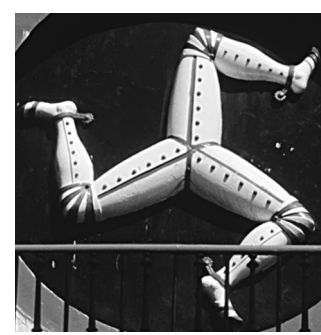
To take a simple example, in the cube below there are 12 edges, 8 vertices, and 6 faces, and $8 + 6 - 12 = \mathbf{2}$.



Euler showed that this formula holds for any polyhedron. Specifically, if the number of edges, vertices, and faces are denoted by e, v, and f, respectively, then $v + f - e = 2$.

(No, the e you see there isn't the e from a calculus course, though it is perhaps worth pointing out that the calculus e is named for Euler, whose extraordinary work will appear many times before this book is through.)

3 [prime]



THE "Three Legs of Man" has been the official symbol of the Isle of Man since the thirteenth century. A possible inspiration was the Sicilian flag, which features the head of Medusa surrounded by three legs. Many European flags, notably the French and Italian flags, consist of three vertical stripes.