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Opening Extract from...

How Many Socks Make a Pair

Written by Rob Eastaway

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1

I don't believe it

You will need: a bathroom mirror, a newspaper, a group of people and some imagination.

Some call it common sense. Others call it intuition, or just 'having a hunch'. Whatever you want to call it, everyone has it to a greater or lesser degree. It's what helps us to understand the world without having to spend too long trying to work things out. And most of the time our intuition serves us very well. But there's a phenomenon that is much loved by mathematicians and scientists, and they call it the counter-intuitive. I have to confess that it is the counter-intuitive bits of maths that I love the best, particularly the examples that seem to clash with our own experience of the world.

In this opening chapter, I want to concentrate on four of my all time favourite examples of the counter-intuitive, each with their own share of Ah, Aha! and Haha.

Let's see how your intuition works.



The mirror

Imagine it's early morning, you are in the bathroom, have just cleaned your teeth and are now inspecting yourself in the mirror. From where you are standing close up to the sink, you can see about half your body – suppose the lowest point that you can see down to in the mirror is your navel.

You now start to step back from the mirror. As you retreat, are you able to see?:

(a) Less of yourself
(b) The same amount
(c) More of yourself.

Think about it for a moment. The answer should be obvious enough, after all you've probably inspected yourself in a mirror thousands of times.

Do you think the answer is (c), that as you step back from the mirror you see more of yourself? That is what most people say. It seems plausible, after all when you try on new clothes in a shop it is normal to step back to get a better view.

It is, however, the wrong answer.

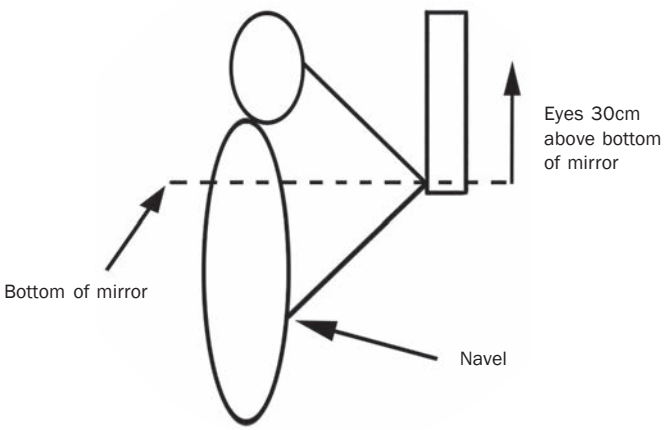
The second most popular answer is (a). As you step back from a mirror you get smaller, so in that sense you see 'less of yourself', though it doesn't answer the question about whether you can now see above or below your navel.

In fact (a) is wrong too. The correct answer is (b) – as you step back from a mirror, the amount of yourself that you can see does not change. This answer comes as a surprise to most people. In fact, it is such a surprise, that it is common for anyone who hears it for the first time to immediately disappear to the bathroom to check it out. You might be tempted to do that yourself right now.

Strictly speaking the answer is only true if the mirror is vertical and the floor is horizontal (which is true of most mirrors and floors, but not the tilting mirrors in shops for example). And of course, you mustn't cheat by leaning over when you get close to the mirror (I've known protestors to lean over and shout 'I can see my toes now!').

The mirror question is a lovely example of something that is counter-intuitive. All our common sense points in one direction, yet in this case it fails us. That feeling of a surprise that you want to check out for yourself is something that can be found throughout mathematics. And having checked it out, for many people that is enough. But for others, maybe you, there is a nagging question: why? And it is when you ask why that you are really 'doing maths'.

In the case of the mirror, the 'why' is easy enough to explain. When you stand close to a mirror, what you are seeing is a reflection of light rays from your body bouncing off the mirror and into your eyes. Here's a diagram showing you close up to the mirror:



Suppose your eyes are 30cm higher than the bottom of the mirror. If you are standing up straight, this means that the lowest point of your body that you can see will be 30cm below the bottom of the mirror. (This is because a light ray bounces off a mirror at the same angle as it meets it.) As you step back from the mirror, the amount that your eyes are above the bottom line of the mirror doesn't change, it is still 30cm. So however far back you stand, you will always see down to your navel.

That, very informally, is the mathematical proof of the mirror question. It may not convince you, it may not interest you, it may still seem to confound your intuition, but it is right. And you can test it, and talk about it. That, to me, is the best sort of maths.

The wedding party

Now that your intuition has been challenged, let's take another everyday example. Imagine you are at a wedding, and there are about 50 guests, young and old. Chatting idly with other guests, somebody says: 'I wonder what the chance is that there are two

people in this wedding party who have the same birthday as each other – not the same birth date, just the same birthday, like 5 May or something.'



You know that there are 365 days in the year. So what is your hunch telling you about the chance of discovering a birthday coincidence in this wedding party of 50 people?

(a) No surprise at all, you think it is almost a dead-cert that two of the people share a birthday.
(b) It feels like it's probably about a '50–50' chance.
(c) You would be very surprised. This would only happen at, say, one wedding in 7.
(d) You would be freaked out. The chance of this must be about the same as that of a rank outsider winning a horse race.

All of us have enough experience of coincidences that happen in our lives to build up some sort of intuition for how likely something is. And yet when it comes to coincidences, our intuition is often way off the mark. If you ask a typical group of adults the birthday question, the majority would be surprised by a coincidence. Most answer either (c) or (d). Some will have

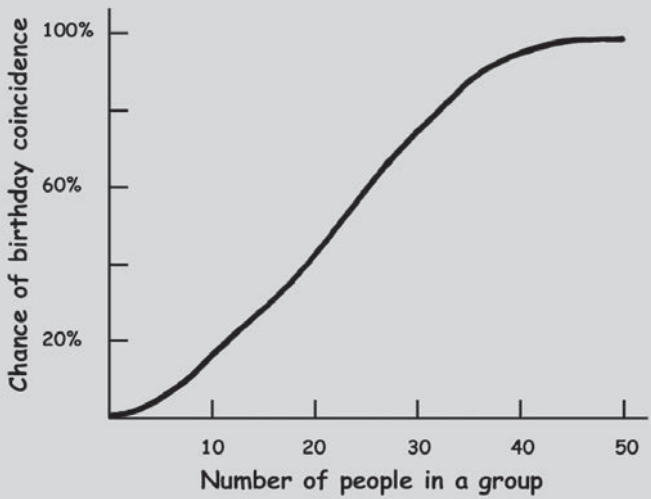
heard about this puzzle before and will remember that 50–50 features in it somewhere, so plump for (b). In fact the correct answer is (a) – with fifty people in a room, it is indeed almost a dead-certainty that there are at least two people in the room who share a birthday. Assuming that birthdays are evenly spread across the calendar, then the chance of a coincidence is about 97 per cent, so you might expect to go to 30 gatherings of this size and only find one where no birthday coincidence happened.

If you want the chance of a birthday coincidence occurring to be 50–50, you only need to have a group of 23 people. The mathematician Robert Matthews thought of a neat way of identifying a 'random' group of 23 people: just take the two teams in any football match, and add in the referee. If you check the birthdays of the players plus the referee in a weekend of fixtures, then you will find that in half the games, two people on the pitch share a birthday.

And in fact the odds are even better than I've said, because birthdays are not evenly spread around the year. Some months are more fertile than others. Typically there are more births in September than in other months (which gives a hint of what must go on at some of those Christmas parties). This clustering increases the chance of a coincidence by another couple of per cent.

Part of the reason why people are so surprised by this result is that we get confused between the chance of two *particular* people having the same birthday (which is a remote 1 in 365) and the chance of any two people sharing a birthday. In a group of 50 people, there are over a thousand different pairs you could choose from the group – Alf and Bert, Alf and Celia, Bert and Celia ... and so on – so perhaps it's not quite so surprising that among all those combinations you find one where the pair go snap.

If you like this sort of thing, you can actually plot out the chance of a birthday coincidence for a given number of people.



With small groups, the chance of a birthday coincidence grows slowly. With 10 people, it is just 12 per cent, but the curve steepens and doubling the number of people to 20 more than trebles the coincidence chance to 43 per cent. By 30 people it is up to 70 per cent, and at 50 people, when the curve starts to flatten, the chance of a birthday coincidence is up to 97 per cent. Yet you can't be 100 per cent certain of a coincidence until there are 367 people in the room (because one might have a 29 February birthday).

But why do you need just 23 people to get a 50–50 chance of a coincidence? How do you actually work it out? There is a difficult way and a (relatively) easy way. The difficult way would be to say: a coincidence means there might be two people with the same birthday, or three, or two different pairs with shared

birthdays, or five people with the same birthday plus a separate threesome. To work out the chance of a coincidence, you could list these (literally) thousands of different scenarios and add up the chance of each one. The alternative approach, which is the one that statisticians always use, is to say: what is the chance that in the group of 23, everyone has a *different* birthday? When we've worked that out, whatever is left must be the chance that there is at least one coincidence in the room.

The chance of a coincidence with 23 people turns out to be $^{365}/_{365}$ x $^{364}/_{365}$ x $^{363}/_{365}$ and so on until you get down to $^{343}/_{365}$, which works out at 0.493, which is near enough 50 per cent.

Do you feel better knowing that? If not, don't worry, we'll move on.

The football stadium

Frimpton United have won the league, and the club decides to hold a reception for the team. The groundsman brightens things up by buying some bunting – flags on a string – to go along the touchline. The idea is that as the players come out of the dressing rooms at the centre of the stand, they will run under the bunting and onto the field. The football pitch is 100 metres long, and the groundsman buys bunting that is 101 metres long. He ties the bunting to the base of the two corner flags on that side of the pitch. The manager spots what he is doing and challenges him. 'You've only got one metre of slack in the bunting!' he exclaims. 'If you lift up the bunting in the middle, is there going to be enough room for the players to get underneath?'



What happens when the bunting is lifted?

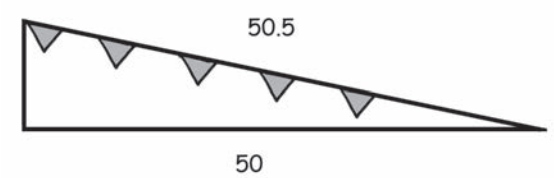
(a) The bunting will be so tight, the players will barely be able to squeeze underneath.
(b) The players will be able to get under the bunting if they crouch very low.
(c) Most of the players will be able to walk underneath without needing to stoop at all.

By now, of course, you smell a rat. This is a chapter about surprises, in which case whichever answer you go for, it's bound to be the other one. So in this three card trick, which do you go for, (a), (b) or (c)? There's one metre of slack, but it has been stretched over a very long distance, so your intuition might well pull you towards (a), that the string can hardly be lifted off the ground at all. But since there is a catch, you go for (b) or (c) instead.

In this case, all three answers are wrong. Not only will all the players be able to comfortably walk under the bunting, but if they get into the team coach, the whole coach can comfortably be driven under the lifted bunting with room to spare. With just a metre of slack in the bunting, the string can be lifted up to a height of 5 metres at the centre of the pitch.

Maybe in this situation, intuition goes wrong because 'sharing' one metre of string along a very long length sounds like sharing five loaves between five thousand people – in other words, you don't expect it to go very far. But that happens not to be the way that the geometry in this problem works. The explanation here is to do with the string forming a very long, thin triangle, and the numbers can be worked out using the ancient theorem of Pythagoras.

Imagine yourself standing at the halfway line. The bunting string and the touchline form two sides of a very long, thin right-angled triangle, like this:



The base of the triangle is 50 metres, and the long side (the hypotenuse) is 50.5 metres. We need to know how high the triangle is, since that's the space through which the players will be passing. You might remember from your youth a rule that says that on a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. It's called the theorem of Pythagoras. Using Pythagoras, it turns out that the height of the triangle is just over 7 metres, or 23 feet, which is considerably taller than a typical double-decker bus.

But again, it's not so much the maths that interests me here, it's the sheer surprise in the answer.

The newspaper statistics

Let's finish with one more everyday example. Indeed, this one is so 'everyday' that it is perhaps the biggest surprise of all.

Tomorrow's newspapers will be filled with stories, many of them involving numbers, and all of them unpredictable. There may be news of job losses, a company may announce record profits, there may be a report about a number of people killed in a natural disaster, and who knows, perhaps news will come in of a record being broken, such as the largest ever number of tooth-picks used to build a model of the Taj Mahal.

But the point is, tomorrow's news is *unpredictable*. Given that these numbers are unpredictable, and that they have no connection to each other, it is hard to imagine that there could be any kind of 'pattern' connecting those numbers. And yet, it is very likely that the numbers *will* form into a pattern.

Pick up any newspaper and look for stories that contain numbers. The sorts of numbers to look for are measurements (height, distance, weight and so on), sums of money (profit, pay, prizes) and statistics that are counting things (number of cars in a pile-up, number of tourists entering the country and so on). You should ignore dates, percentages, people's ages and telephone numbers, for reasons that will be explained shortly.



Find 10 numbers from different parts of the newspaper, and write them down.

Since these numbers are effectively random, the first digit of each number will be 1, 2, 3, 4, 5, 6, 7, 8 or 9. And you might suppose that a number, such as (say) the total visitors at Disneyland last month, is just as likely to begin with a 1 as with a 9. But that

is not the case. In fact, the number visiting a randomly selected theme park like Disneyland[1] on any particular day, is extremely likely to begin with a 1, a 2 or a 3, and extremely unlikely to begin with an 8 or a 9. Of the statistics that you have selected at random from today's newspaper, about half will begin with a 1 or a 2, and I'm prepared to bet that you didn't pick any that began with 8 or 9. (You will find them, but they are very rare.)

This remarkable phenomenon is known as Benford's Law. It applies to any selection of statistics, so long as the numbers being chosen are counting something that could be as low as zero, and have no obvious upper limit. Redundancies, numbers injured and sales figures can be just about any number, and generally obey Benford's Law.

However, the ages of people in the obituary column don't obey Benford's Law. The vast majority of people listed in the obituaries are aged between 60 and 90 because (a) the older you are the more likely you are to have done something worthy of an obituary in your life, and (b) because typically the vast majority of the people who die on any given day are aged between 60 and 90. Ages, particularly obituary ages, do not obey Benford's Law. Nor do other constrained numbers including percentages (these will tend to be uniformly spread between 0 and 100) and telephone numbers (these are labels rather than numbers, and do not obey normal statistical laws – for example in London most numbers begin with an 7 or 8).

As with all of the other counter-intuitive examples in this chapter, Benford's Law invites the question: why? How does a mountain 'know' that its height should begin with a 1 far more often than a 9? And even more spookily, how come this works whether we decide to measure the height of the mountain in

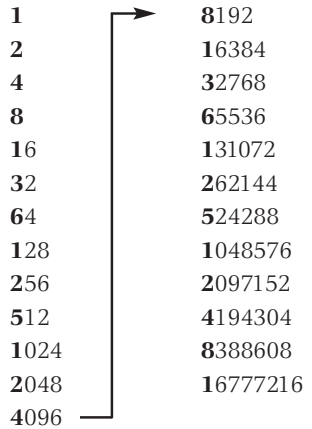
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[1] I should be careful to point out here that I've picked out Disneyland at random, I know nothing about the attendances at Disneyland, Alton Towers or any other theme park.

centimetres, feet or broomsticks? How does an earthquake 'know' that the number of victims will follow a similar pattern?

This is when the power of mathematics can begin to really get under your skin, because it turns out that fundamental mathematical patterns are at work that dictate that Benford's Law will be so. The detailed explanation as to why Benford's Law works is rather involved, and I have no intention of going into it here. But I should at least give a hint of where it comes from. It is related to the fact that the world is full of things that grow 'exponentially'.

A simple example of exponential growth is if you take the number 1 and keep doubling it. The first 25 numbers you get when doubling are:



Notice anything about the leading digits? So far, there have been eight leading 1s, but we have yet to encounter any 9s. Part of the reason why this is the case is that in order to get a leading digit of 9, the previous number that you are doubling has to begin 45..., 46..., 47..., 48..., or 49... . (If you don't believe me, check it out on a calculator). On the other hand, if you double any number that begins with 5, 6, 7, 8 or 9, the answer begins with a 1.

So Benford's Law works for numbers that are forever doubled, and in fact it is also true for numbers that grow by any other fixed proportion, otherwise known as exponential growth.

You've just seen that things as ordinary as a mirror, a birthday, a piece of string and a newspaper can contain jaw-dropping surprises. And all of them can be completely explained by maths. I hope it has aroused your curiosity.